Rania Hegab

Exam II: MTH 111, Spring 2018

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Points = $\frac{55}{47}$

55/55 Excellent !!

QUESTION 1. (8 points) Find y' and DO NOT SIMPLIFY

(i) $y = 6e^{(3x^2+6x+1)}$

$\boxed{y' = 6e^{(3x^2+6x+1)} \cdot (6x+6)}$ ✓

(ii) $y = (2x+3)\sqrt{7x+2}$

$y = (2x+3)(7x+2)^{\frac{1}{2}}$

$y' = (1)'(2) + (2)'(1)$

$\boxed{y' = 1(7x+2)^{\frac{1}{2}} + \frac{1}{2}(7x+2)^{-\frac{1}{2}}(2x+3)}$ ✓

(iii) $y = ln[\frac{(3x+2)^3(2x+7)^2}{(7x+12)^4}]$

$y = 3\ln(3x+2) + 2\ln(2x+7) - 4\ln(7x+12)$

$y' = \frac{3(3)}{3x+2} + \frac{2(2)}{2x+7} - \frac{4(7)}{7x+12}$

$\boxed{y' = \frac{9}{3x+2} + \frac{4}{2x+7} - \frac{28}{7x+12}}$ ✓

(iv) $y = 2(3x^2+5x)^{12}$

$\boxed{y = 24(3x^2+5x)^{11} \cdot (6x^1+5)}$ ✓

QUESTION 2. (i) **(3 points)** What can you say about the line $L : x = 2t+1, y = t-1, z = -2t+3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?

L: $x = 2t + 1$
$y = t - 1$
$z = -2t + 3$

P: $x + 2y + z = 16$

$(2t+1) + 2(t-1) - 2t + 3 = 16$

$2t + 1 + 2t - 2 - 2t + 3 = 16$

$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$ ✓

x: $2(7)+1 = 15$
y: $7 - 1 = 6$
z: $-2(7)+3 = -11$

$\boxed{\text{Q: intersection point}: (15, 6, -11)}$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N = \langle -2, 3, 2 \rangle \perp P$ at $Q(-1, 4, 2)$

Find eqn → Directional vector
→ point Q

P: $-2(x+1) + 3(y-4) + 2(z-2) = 0$

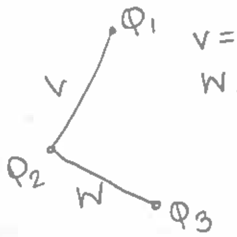
P: $-2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$ ✓

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of plane → directional vector and point Q_1

$Q_1 : (4, 4, 0)$
$Q_2 : (0, 2, 6)$
$Q_3 : (4, 0, 8)$



$V = Q_1Q_2 = \langle 4, 2, -6 \rangle$

$W = Q_3Q_2 = \langle 4, -2, 2 \rangle$

$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, - \begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix}$

$= \langle 4 - 12, -(8 + 24), -8 - 8 \rangle$

$= \langle -8, -32, -16 \rangle$

P: $-8(x-4) - 32(y-4) - 16(z+0) = 0$

P: $-8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$ ✓

QUESTION 3. (i) **(4 points)** (1) Convince me that the line $L: x = 4t, y = -4t+1, z = 2t+1$ is perpendicular to the plane $P: 2x + -2y + z = 12$ (If you think that I am wrong, then state your reason). (2) Can we draw the vector $V = <1, -2, -6>$ inside P?

L: $x = 4t$
$y = -4t + 1$
$z = 2t + 1$

P: $2x + -2y + z = 12$

$D_2 = <2, -2, 1>$

(1) $D_1 = <4, -4, 2>$

$$D_1 \times D_1 = \begin{vmatrix} i & j & k \\ 4 & -4 & 2 \\ 2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 2 \\ -2 & 1 \end{vmatrix}, - \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 4 & -4 \\ 2 & -2 \end{vmatrix} = -4+4, -(4-4), -8+8$$

$\boxed{D_1 \times D_2 = <0, 0, 0>}$ → $\boxed{\text{Plane and line are perpendicular.}}$

(2) $V = <1, -2, -6>$

$D_2: <2, -2, 1>$

$V \cdot D_2 = 0 \to$ Yes $V \cdot D_2 \neq 0 \to$ No

$V \cdot D_2 = <1, -2, -6> \cdot <2, -2, 1>$

$\boxed{V \cdot D_2 = 2 + 4 - 6 = 0 \to \text{Yes, we can draw V inside P.}}$

(ii) **(3 points)** Find the distance between $Q = (10, 10, 33)$ and the plane $P: 2x - 2y + z = 21$.

$Q: (10, 10, 33)$

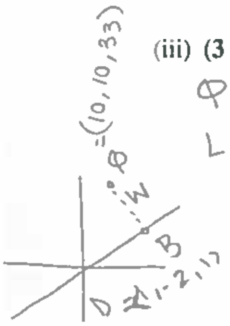
$P: 2x - 2y + z = 21$

$\to 2x - 2y + z - 21 = 0$

$$QP = \frac{|2(10) - 2(10) + 33 - 21|}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$QP = \frac{12}{3} = \boxed{4 \text{ units}}$$

(iii) **(3 points)** Find the distance between $Q = (10, 10, 33)$ and the line $L: x = t+1, y = -2t+3, z = t$



$Q: (10, 10, 33)$

L: $x = t + 1$
$y = -2t + 3$
$z = t$

$D = <1, -2, 1>$
$I = <1, 3, 0>$

$W = |QB| = 9, 7, 33$

$$QL = \frac{|W \times D|}{|D|} \quad |W \times D| = \begin{vmatrix} i & j & k \\ 9 & 7 & 33 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 33 \\ -2 & 1 \end{vmatrix}, - \begin{vmatrix} 9 & 33 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 9 & 7 \\ 1 & -2 \end{vmatrix}$$

$= <7 + 66, -(9-33), -18-7> = <73, 24, -25$

$$\frac{|W \times D|}{|D|} = \frac{\sqrt{73^2 + 24^2 + 25^2}}{\sqrt{1^2 + 2^2 + 1^2}} = \boxed{32.99 \text{ units}}$$

(iv) **(6 points)** The two planes $P_1: x + 2y + z = 10$ and $P_2: -x + 2y - z = 6$ intersects in a line L. Find a parametric equations of L.

$P_1 = x + 2y + z = 10 \to N_1 = <1, 2, 1>$

$P_2: -x + 2y - z = 6 \to N_2 = <-1, 2, -1>$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}, - \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$$

$= <-2-2, -(-1+1), 2+2>$

$\boxed{N_1 \times N_2 = <-4, 0, 4>}$

Let $\boxed{z = 0}$ in P_1 and P_2

$x + 2y = 10 \to x = 10 - 2y \to 10 - 2(4) = 10 - 8 = \boxed{2 = x}$

$-x + 2y = 6$

↓

$-(10 - 2y) + 2y = 6$

$-10 + 2y + 2y = 6$

$-10 + 4y = 6$

$4y = 6 + 10$

$4y = 16 \Rightarrow y = 16/4 \Rightarrow \boxed{y = 4}$

Parametric eqns:

$x: -4t - 2$

$y: 4$

$z: 4t$

QUESTION 4. (7 points) Let $f(x) = -x^3 + 6x^2 + 15x + 1$.

(i) For what values of x does $f(x)$ increase?

$f'(x) = -3x^2 + 12x^1 + 15$

$\boxed{\begin{array}{l} x = 5 \\ x = -1 \end{array}}$

$f(x)$ increases $\rightarrow (-1, 5)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ decreases $\rightarrow (-\infty, -1) \cup (5, +\infty)$

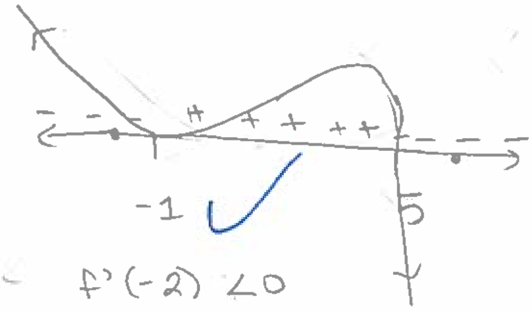
(iii) Find all minimum, maximum points of $f(x)$.

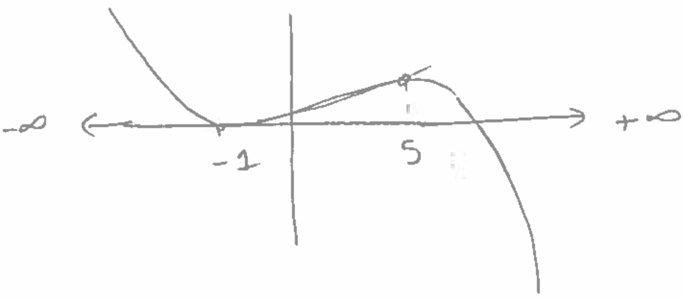
min at $x = -1 \rightarrow$ $(-1, -7)$

max at $x = 5 \rightarrow$ $(5, -42) \rightarrow 101$

$-(5)^3 + 6(25) + 15(5) + 1 =$ 101

(iv) Roughly, sketch the graph of $f(x)$.





$f'(-2) < 0$

$f'(0) > 0$

$f'(6) < 0$

QUESTION 5. **(4 points)** Let $f(x) = 2xe^{(x-1)} + ln(2x-1) + 4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$F(x) = 2xe^{(x-1)} + \ln(2x-1) + 4$

$Q: (1, F(1)) = (1, 6)$

$f(1) = 2(1)e^{(1-1)} + \ln(2(1)-1) + 4 = 6$

$f'(x) = (1)'(2) + (2)'(1) + \dfrac{\log(2x-1)}{\log(10)} + 0$

$f'(x) = 2e^{(x-1)} + e^{(x-1)}(1)(2x) + \log(2x-1) \cdot 1/\log_{10}$

$f'(x) = 2e^{(x-1)} + 2xe^{(x-1)} + \dfrac{2}{\log(10)} \Rightarrow f'(1) = 6$

$y = mx + b$

$6 = 6(1) + b$

$6 = 6 + b$

$6 - 6 = b$

$b = 0$

$\boxed{y = 6x}$

QUESTION 6. **(7 points)** Consider $f(x) = 4 - \sqrt{x}$, $k(x) = -2$. Find the length and the width of the largest rectangle that you can draw between $f(x)$ and $k(x)$, see picture.

Faculty information

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$\rightarrow A = l \cdot w$

$A = m(6 - \sqrt{m})$

$A = m(6 - m^{1/2})$

$A = 6m - m^{3/2}$

$\rightarrow A' = 6 - \frac{3}{2} m^{1/2}$

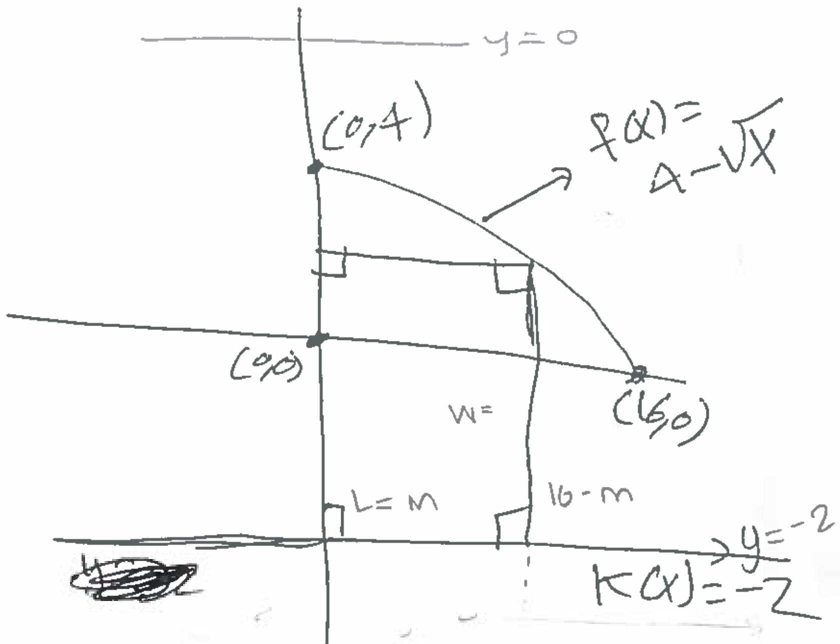
$\rightarrow 0 = 6 - \frac{3}{2} m^{1/2}$

$6 = \frac{3}{2} m^{1/2}$

$\frac{6}{3/2} = \frac{3/2}{3/2} m^{1/2}$

$0.5\sqrt{4} = 0.5\sqrt{m^{1/2}}$

$\boxed{M = 16}$



$L = m$

$W = 4 - \sqrt{m} + 2 = 6 - \sqrt{m}$

$L = m = \boxed{16}$

$W = 6 - \sqrt{m} = 6 - \sqrt{16} = \boxed{2}$

$\rightarrow A'' = -\frac{3}{4} m^{-1/2}$

$A''(16) = -\frac{3}{4}(16)^{-1/2} < 0 \rightarrow \boxed{\text{max.}}$